

# Modelling NMDs - A Review

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4th May 2021

## Abstract

In this article we are going to review the modelling of NMDs via replicating portfolios due to the revived interest in NMDs in the context of the interest rate risk of the banking book (IRRBB). The main goal is to provide a self contained presentation of the replicating portfolio approach from scratch. It intends to clarify the underlying assumptions and the methodology of the replicating portfolio approach, i.e. it derives the theory from simple basic principles while collecting all relevant information in one place. Because using this model is a major methodological decision we will pay particular attention to the challenges this modelling approach is exposed to in a low interest environment, which is characterised by a pronounced regime switch with respect to the interest rates of the eligible investment products.

**Keywords:** NMD, replicating portfolio approach, low interest rates, IRRBB, BCBS368

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# 1 Introduction

Non-maturing deposits, i.e. a subclass of products with undefined maturity, are of special importance to commercial banks because they usually contribute a significant part to the balance sheet on the liability side. These products are characterised by the following properties:

- The product is a daily callable deposit, characterised by a balance and an interest rate.
- The balance of the deposit can be increased or decreased by the depositor at any time without having to pay any compensation to the bank.
- The bank can adjust the interest rate paid to the depositor at any time.

Changes of the product's interest rate are made due to opportunity considerations by the bank in competition with other banks. If the market-wide level of interest rates changes, interest rate adjustments are not carried out instantaneously, but typically with a delay. The level of the interest rate of the NMD is set with respect to a stable amount of NMDs on the liability side.

The main assumption of the replicating portfolio approach is the existence of a more or less stable amount of NMDs on the liability side of the balance sheet whose rate is determined mainly by competition with other banks and not directly by the current level of market interest rates thus leading to the aforementioned lagged interest rate adjustment. The existence of this stable amount is a purely statistical effect [1]. Each inflow of money at a specific point in time does decay stochastically because money is spent earlier or later. Yet, when considering the superposition of a large number of in- and outflows of a large number of debtors, all contributing to the total balance (overall current balance), then it turns out to be possible to define a certain part of the total balance that is permanently present with a high level of confidence. The fluctuations above the stable part form the volatile part, which is the complement of the stable part with respect to the total balance. The separation into a stable and a volatile part is a topic of a statistical analysis and not part of the discussion we are going to unfold here (for more details cf. [2]). Historically the replication approach was developed to model only the core ('Bodensatz'), i.e. that fraction of the stable part that is unlikely to reprice even under significant changes in the interest rate environment. Later it was extended to apply to the full stable part and to account for occasional changes in the volume of the stable part, too. For the sake of completeness, a concise introduction into the topic of NMD modelling was given in [3] and a review of the existing scientific literature can be found e.g. in [4].

The rest of the paper is organized as follows: In Section 2 we consider the stable part only and describe the general methodology to replicate the stable part in terms of a portfolio of bonds. Later, in Section 3 we will discuss the differences between the stable part and the core in more detail and argue that in the prevailing market regime of interest rates the main assumption on the

modelling approach is violated. Finally, in Section 4 we summarise the main conclusion for the modelling of NMDs via replicating portfolios that follow from the existence of two very different market regimes of interest rates.

## 2 Replicating Portfolio Approach

The replicating portfolio approach is a hypothetical practice of investing NMD volumes into term deposits (replicating portfolio) according to a predetermined disposition rule.

Before delving into the details of this approach, we will introduce the concept of a decay function (or run off profile) as a tool to describe the run off of the total NMD volume invested into a replicating portfolio, when one artificially stops (re)investing NMD money into the replicating portfolio (dead portfolio assumption). In addition we will present a specific technique to approximate the decay function in terms of a convex combination of a set of linear amortising run off profiles<sup>1</sup>.

For illustration purposes in figure 1 the run off of the stable part of the total volume invested in a replicating portfolio according to a deterministic decay is displayed. In addition two linear amortising profiles are displayed that are chosen to match the 'true' decay function as close as possible. The first profile amortises over three months and the second profile over ten years. The two positive attachment points  $\omega_{3M}$  and  $\omega_{10Y}$  define a positive set of weights that add up to 1. These weights can be rescaled with the total invested volume.

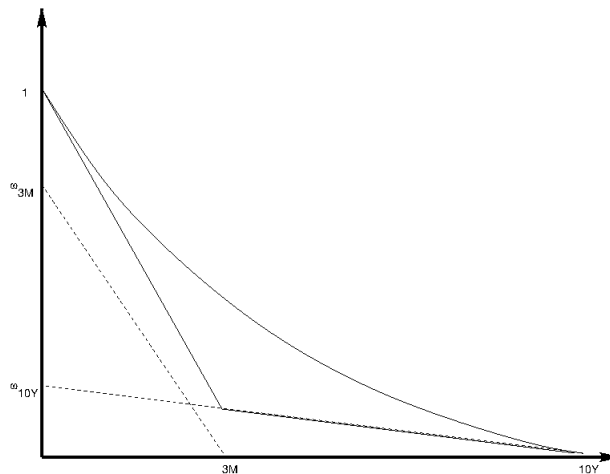


Figure 1: Approximation of a decay function via a convex combination of two linear amortising profiles.

<sup>1</sup>Linear amortising run off profiles are chosen because the approximation in terms of a disposition rule of actual trades is possible. As will be shown in Section 2.1, the continuous reinvestment in bonds of the same maturity preserve the linear amortising run off profile over time.

Obviously the approach in this example can be generalised, such that any reasonable decay function can be approximated by a convex combination of linear amortising profiles. In principle the number of convex combinations of linear functions can be extended by any other linear profile, that amortises over any sensible period of time. In practice the maximal maturities of the linear amortising profiles are limited by 10 years, for instance. Let us assume that  $k$  linear amortising profiles are superposed with  $k$  positive attachment points  $\omega_{i \in \{1, \dots, k\}}$  subject to the condition  $\sum_{i=1}^k \omega_i = 1$ . Then the convex combination of the corresponding linear amortising profiles defines the approximation of the decay function used to describe the run off of the replicating portfolio connected with the stable part. One important point to note is that the above idea of describing the decay of the replicating portfolio connected with the stable part does not need much more information than the mere balance and its allocation onto the separate linear amortising profiles as defined by the positive weights  $\omega_i$ . If we look for any technique, which is used to implement this idea in terms of a portfolio of tradeable market instruments, the linear amortising profiles must be realised in terms of the notionals of these replicating instruments. In particular, up to now interest rate payments do not play a distinguished role in the determination of the decay function as a superposition of linear amortising profiles<sup>2</sup>. In principle any type of liquid market instrument with a single notional repayment at maturity is suitable to do the replication. Examples of possible alternatives are fixed rate bonds, floaters and constant maturity bonds (CMB). For each of these alternatives (synthetic) market prices can be derived (at least from liquid markets of derivatives). The main differences between the different choices of replicating instruments are the calculation of FTP rates and the sensitivity structures associated with the replicating instruments<sup>3</sup> (see [2, 5]).

We will explain in the next section how the replicating portfolio approach which is using fixed rate bonds as replicating instruments can be used to realise an approximated realisation of the ideas described so far.

## 2.1 Mathematical Basis

The replicating portfolio approach uses e.g. fixed rate bonds to realise linear amortising profiles, i.e. the building blocks in modelling the decay function described before. The idea behind this reformulation of the linear amortising profile in terms of actual market instruments mimics the practice of a treasury department, that money which is received from a depositor in form of (the stable part of) an NMD is reinvested at current market rates into e.g. fixed rate bonds. This is called a disposition rule. The purpose of this section is therefore to describe the disposition rule, which realises a discretised version of the approximation of the true decay function of Figure 1 in terms of the convex combination of linear amortising profiles.

<sup>2</sup>In Section 2.2 we will see, that also interest payments may play an indirect role in determining a decay function.

<sup>3</sup>The choice of the replicating instrument is mainly driven by the structure of the active side of the balance sheet.

In this section we proceed as follows. First, the breakdown of the problem onto disposition rules that match the elementary linear amortising profiles is described. Then the actual disposition rule is formulated as a simple formula. After having achieved this, it is shown that the same formula can be used to handle the initialisation of the replicating portfolio as well as the treatment of the situation, where the total balance increases, decreases, or remains constant.

## 1. Breakdown to Linear Amortising Profiles

All considerations start from an overall current balance  $V(0) := V(t)|_{t=0}$ . Due to the positive weights  $\omega_{i \in \{1, \dots, k\}}$  introduced before, it is possible to assign volumes to each element in the set of linear amortising profiles with maturity  $T_{i \in \{1, \dots, k\}}$ . The corresponding profile specific volumes are defined as  $V_i(0) = \omega_i \cdot V(0)$ .

## 2. The Building Blocks – Realisation of Linear Amortising Profiles via Bonds

The final prerequisite for the description of the replication algorithm is to link linear amortisation profiles to a set of market instruments (fixed rate bonds). For simplicity we discuss the example of a linear amortisation profile with a maturity of 10 years (such as the representative displayed in Figure 1). A bullet bond does not possess an amortisation profile of the notional itself, thus the aim of mimicking the linear amortising profile is achieved by applying a trick. The trick can be easily described by thinking in terms of a set of bonds which all are considered from the same reference date, but their residual maturities  $\{T_i^{(l)}\}_{l=1 \dots i}$  grow from a shortest maturity  $T_i^{(1)}$  until the largest maturity  $T_i^{(i)} = T_i$  (here 10 years) with a regular spacing according to a preselected frequency<sup>4</sup>. The preselected frequency is the same frequency at which interest payments occur in the NMD and thus defines also the value of the shortest maturity.

$$T_i^{(1)} \leq T_i^{(2)} \leq \dots \leq T_i^{(i-1)} \leq T_i^{(i)} = T_i$$

If the profile specific volume  $V_i(0)$  is uniformly distributed over all those bonds, then an approximated linear amortising profile can be retrieved, when one considers the total notional invested in bonds with a maturity larger than  $t$ , i.e.

$$V_i(t) = V_i(0) \cdot \sum_{T_i^{(l)} > t} N_i^{(l)} / V_i(0),$$

where  $T_i^{(l)}$  is the maturity and  $N_i^{(l)}$  the notional invested in the  $l$ -th bond of the above sequence used to represent the  $i$ -th linear amortising profile. Obviously all the  $N_i^{(l)} / V_i(0)$  are equal and sum up to 1. This is displayed in Figure 2.

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<sup>4</sup>These maturities  $T_i^{(l)}$  have to be understood as relative to the periodically moving reference date.

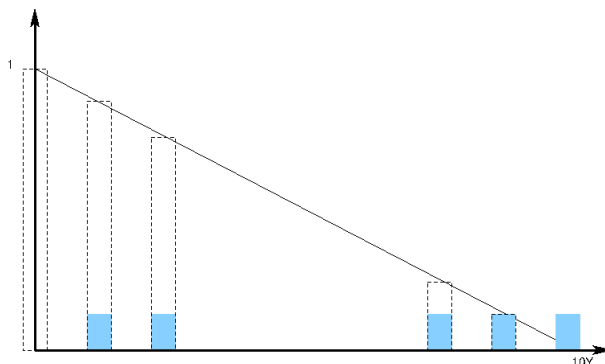


Figure 2: Representing a linear amortising profile in terms of a disposition rule. The blue bars indicate the volumes  $N_i^{(l)}/V_i(0)$ , while the dashed bars represent the cumulative sum that is approximating the linear amortising profile.

If the disposition rule is applied only to set up the portfolio and nothing else is done henceforth, then at time zero the total notional is bound in the set of bonds with the same starting dates and maturities  $T_i^{(l)}$ . At all successive future dates specified by the repayment frequency equal parts of the initial invested volume are repaid to the depositor according to the assumption of linear amortization. This in particular means, that the remaining invested value amortizes linearly down to zero at  $T_i$ .

Traditionally it is said, that the part of the NMD balance  $V_i(0)$  is invested into a “rolling” structure. That means, the part of the volume which is periodically repaid is instantaneously reinvested into a bond with residual maturity  $T_i$  at current market rate, thus preserving the linear amortizing profile [2, 6]. The current market rate corresponds to the maturity matched market interest rate (FTP rate) as of the date of investment.

In case a change in volume of  $V_i(t)$  at some future point in time  $t$  is taken into account, this additional amount is again equally distributed to a number of bonds with residual maturities  $T_i^{(l)}$ , thus preserving the profile. The investment occurs at the current market interest rate for the respective maturities. This will be formalised, now.

### 3. General formula for the disposition rule

The additional volumes  $\Delta V_i^{(k)}(t)$  of replicating bonds with maturity  $T_i^{(k)}$  belonging to the representation of a linear amortizing profile with maturity  $T_i$ , which must be traded at current market conditions due to the disposition rule are given by<sup>5</sup>:

<sup>5</sup>Please note, that now the weights are made time dependent as well. This should be understood as a step function, i.e. during a certain period the  $\omega_i(t)$  always stay constant.

$$\Delta V_i^{(k)}(t) = \begin{cases} \frac{V(t) \cdot \omega_i(t) - V(t-1) \cdot \omega_i(t-1)}{n_i} & \text{if } T_i^{(k)} < T_i \\ \frac{V(t) \cdot \omega_i(t)}{n_i} & \text{if } T_i^{(k)} = T_i \\ 0 & \text{if } T_i^{(k)} > T_i \end{cases} \quad (1)$$

Here  $n_i$  is a number obtained by dividing the number of months corresponding to the maturity  $T_i$  by the preselected (interest payment) frequency also expressed in months. In the special but important case of monthly replication,  $n_i$  turns out to be simply the number of months until  $T_i$ . This formula can be applied to all cases of importance. Two special situations worth mentioning are:

1. The initialization of the portfolio ( $V(t-1) = 0$ ,  $\omega_i(t-1) = 0$ ) and
2. the case where neither volume nor weights change ( $V(t) = V(t-1)$ ,  $\omega_i(t) = \omega_i(t-1)$ ).

The second case corresponds to the classical technique of weighted moving averages described in [7].

#### 4. Computing averaged FTP of a replicating portfolio

The interest amount (IA) earned on the replicating portfolio can be obtained by summing up the interest amounts earned by each of the bonds the replicating portfolio is made of. Each bond is equipped with a coupon equal to the FTP rate as computed on the day of its inception<sup>6</sup>. Thus, the interest amount of the  $l$ -th bond belonging to the  $i$ -th linear amortizing profile and originally traded at time  $t_h$  is computed as  $IA_i^{(l)}(t_h) = \Delta V_i^{(l)}(t_h) \cdot FTP_i^{(l)}(t_h) \cdot \tau$ . Here  $FTP_i^{(l)}(t_h)$  denotes the FTP rate of the  $l$ -th bond belonging to the  $i$ -th linear amortizing profile and originally traded at time  $t_h$  and  $\tau$  is the length of the accrual period at which payments occur. Then an averaged FTP rate of the replicating portfolio reads:

$$\overline{FTP} = \frac{\sum_h \sum_i \sum_{l=1}^i IA_i^{(l)}(t_h)}{V(t) \cdot \tau}. \quad (2)$$

The computation of a margin requires the comparison of interest earned on the NMD with interest earned on the replicating portfolio. Thus the margin spread of a NMD can be defined as the difference between the interest rate associated with the NMD and the averaged FTP defined above.

<sup>6</sup>The FTP rate is the term rate, only.



## 2.2 Choosing the Profile

After having described the methodology to model NMDs with the replicating portfolio approach, there is still one question left open: Which decay function (run off profile) should be chosen to reflect the behaviour of a particular type of NMD correctly? Answers to that question point at least into two different directions.

1. First, one could argue that a run off profile must be given by some sort of expert judgment. In this view, the profile is nothing else than a tool to encode the opinion of e.g. the treasury department on the drawing behaviour of NMD volumes in the *future*. This corresponds to an assumption on the effective 'capital commitment'.
2. Second, one could make the point that the decay function should explain the dynamics of the lagged adjustment of the NMD rate as close as possible. In order to implement this view one has to rely on *historically observable data* and one has to make the additional assumption that the run off profile possesses a time translation invariance (up to a rescaling due to a changed overall balance). It is this assumption that it most often made in practice.

Both approaches have their pros and cons and work best for the core. As soon as a strong interest rate dependence of NMD volumes comes into play neither of the two approaches have a sound theoretical basis. Therefore, independent of the final approach taken, a regular validation of the model assumptions must be carried out that the risk of a hidden model defect can be controlled appropriately.

### Direction 2: Calibration based on historical data:

In order to identify plausible disposition rules for a specific type of NMDs, one can use an optimization algorithm based on historical data that comprise product volumes, product interest rates and market interest rates [8, 9]. It is important to understand that the application of such an algorithm produces only a self-consistent result, i.e. during the optimization that decay function is determined by the algorithm, which is most consistent with the historical data in the sense of the chosen target function. The quality of the estimated decay function must be regularly supervised by backtesting the model predictions.

The two most common target functions<sup>7</sup> used in practice are [11]

1. the lowest variance of the modelled margin and
2. the highest modelled margin at the lowest risk. The risk is measured in terms of the standard deviation of the margin.

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<sup>7</sup>A good introductory account into the background of the meaning of the two listed target functions can be found in [10].

The outcomes of the optimization algorithm is the set of positive weights,  $\omega_{i \in \{1, \dots, k\}}$ , for the linear amortizing profiles realised in terms of fixed rate bonds. These in turn determine the cash flow structure of the replicating portfolio.

### 3 Discussions of Market Regimes

In this section we are going to discuss the applicability of the replicating portfolio approach in different market regimes. While the approach was developed under “normal” market conditions it has to cope with an expressed low interest rate environment, nowadays. In order to explain the challenge the approach is facing, consider an economy consisting of two segments - banks and individuals - only [12]. Furthermore, suppose there are only two types of products traded in that economy:

1. Standardised term deposits (TD) with different tenors  $\tau_i$ 
  - (a) Banks can both buy and sell TDs.
  - (b) Individuals can only buy TDs (i.e. we disregard the possibility of banks symmetrically trading money with individuals).
2. Non-maturing deposits (NMD) (aka sight deposits)
  - (a) Banks only sell NMDs to individuals, not to other banks.
  - (b) Individuals can only buy NMDs.

The two segments can thus be characterised as follows:

- Banks that have full access to the ‘capital market’ but in addition fund themselves by selling NMDs to individuals, too
- Individuals possessing a limited amount of cash  $D(t)$  that they can invest either in TDs or NMDs

Denote the current interest rate for a TD with tenor  $\tau_i$  as  $r(t, \tau_i)$ . In particular, the O/N rate for a TD with tenor 1 day is denoted as  $r(t, 1d) := r(t)$ . The current interest rate on NMDs is denoted as  $i(t)$ .

The part of the total amount of cash  $D(t)$  available to individuals that is invested in NMDs is denoted as  $D_{NMD}(t)$  while the parts invested in TDs of tenor  $\tau_i$  is denoted as  $D_i(t)$ . We assume that

$$D(t) = D_{NMD}(t) + \sum_i D_i(t),$$

i.e. all the cash of individuals is always invested in either NMDs or TDs. The capital invested in NMDs can be split into three parts, i.e.  $D_{NMD}(t) = D_V(t) + D_C + D_I(t)$ , which are described below:

- A time-dependent volatile part  $D_V(t)$  and a time-independent core part  $D_C > 0$  with  $D(t) > D_V(t) + D_C > 0$ : All individuals constantly deposit and withdraw cash (salaries vs. payments). The volume of cash in NMD accounts due to this process is not related to market prices as changes are not driven by investment decisions, i.e. the crucial characteristic of  $D_V(t)$  and  $D_C$  is that they are entirely independent of  $r(t)$  and  $i(t)$ .
- A time-dependent investment contribution  $D_I(t)$ : This is all the excess cash of individuals invested in NMDs that is not needed for ensuring day-to-day liquidity.

How this split is actually performed will not be discussed here.

Note that in the proposed economy we usually do not observe  $r(t) = i(t)$  (even though O/N deposits and NMDs are technically very similar instruments). Instead one finds two major regimes:

- “normal” market environment, i.e.  $i(t) < r(t)$  is an incentive for individuals to invest in short-term TDs rather than NMDs and one expects  $D_I(t) \approx 0$ . To what extent  $r(t)$  can then be larger than  $i(t)$  depends on the nature of competition for NMDs across banks.
- low interest environment:  $i(t) > r(t)$  is clearly an incentive for individuals to invest cash in NMDs rather than short-term TDs. Thus one expects  $D_I(t) > 0$ . The reason for the existence of this regime is, that the corresponding arbitrage opportunity cannot be exploited neither by individuals (as they cannot short TDs) nor by banks (as they cannot buy NMDs).

The replicating portfolio approach is a popular choice for producing an independent forecast for the development of  $i(t)$  in the future. By representing the core part of the NMD balance as an amortising portfolio of TDs, one effectively assumes a delayed response of  $i(t)$  to shifts of the yield curve. The challenge of this approach is that it relies entirely on the correct identification of the core part  $D_C$  of the total NMD balance  $D_{NMD}(t)$ . Again, we can distinguish the following situations:

1. “normal” market environment, i.e.  $i(t) < r(t)$  is always guaranteed: the investment contribution  $D_I(t)$  to the NMD balance can safely be assumed to vanish and  $D_{NMD}(t)$  is basically independent of market movements. A replicating portfolio approach that simply tries to identify a fluctuating volatile part (replicated in ON TDs) and replicates the stable and presumably roughly constant rest of the balance as long-term TDs is a sensible approach (there is still the problem that competition for NMDs between different banks can lead to changes in  $i(t)$  inconsistent with the replication approach chosen by a particular bank).
2. low interest environment, i.e.  $i(t) \gtrsim r(t)$  occurs: the investment volume  $D_I(t)$  is expected to be significantly non-zero and depends on market movements. It usually contributes to the stable part, but the identification of the core part requires far more care.

Note that while the first scenario is the historically observed case, which led to the development of the replicating portfolio approach in the first place the second scenario is the currently prevailing one (cf. figure 3).

In scenario 2, significant care needs to be taken in order to distinguish the additional investment part  $D_I(t)$  from the core part of the NMD balance (cf. also [13]). Even though, depending on the market environment,  $D_I(t)$  can have a slowly varying form much like the core part, one should be at least cautious to replicate it in the same long-term TDs. This is so as the investment does not have the same structural long-term character as the core part. As soon as  $i(t) \gtrsim r(t)$  does not hold anymore, the investment part might be withdrawn and reinvested in TDs. A simple consistent prescription for a replication approach would be to replicate  $D_I(t)$  in O/N deposits just like the volatile part  $D_V(t)$ . This is actually the proposal of [14].

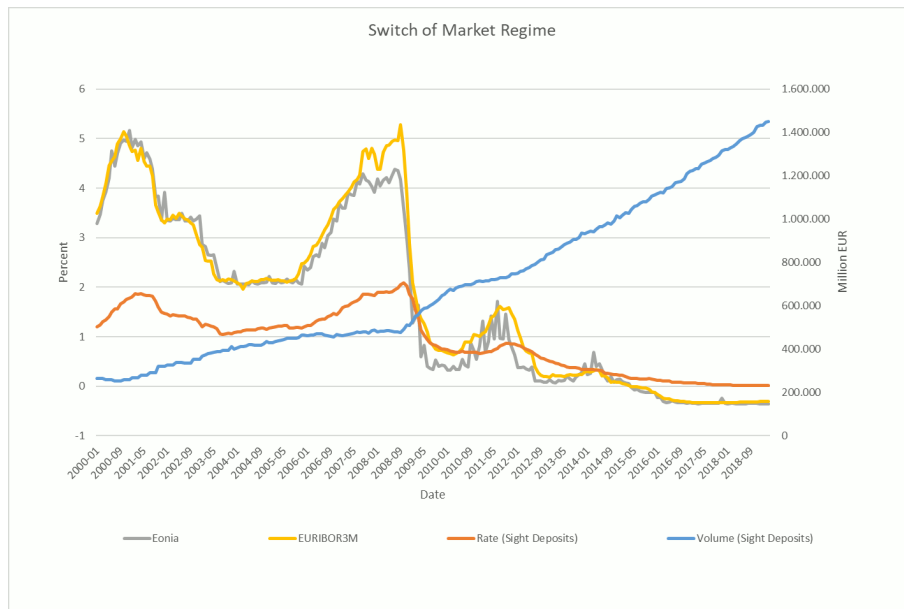


Figure 3: Illustration of the regime change based on Bundesbank statistics. The volume history of daily callable sight deposits of private customers is taken from BBK01.SUD201 and the corresponding rate history is taken from BBK01.SUD101. The data before 2003 are completed using the data from BBK01.SUD101S and BBK01.CEFN0J.

## 4 Summary

The discussion on the behaviour of NMDs in different market regimes has revealed that in the current low interest rate environment the applicability of the replicating portfolio approach requires special care. The reason is simply the

violation of one of its central assumptions: After 2007 the stable part started to differ significantly from the core part and has become more and more interest rate sensitive. This effect is a consequence of the shift in the interest rate regimes, where any reasonable market rate finally ended up to earn less interest than a NMD. As a consequence the nature of the NMD products has changed profoundly. They have become investment opportunities and their interest rate becomes strongly dependent on the level of benchmark market rates. This is actually opposite to the assumptions made on NMDs in the introduction. Thus even a significant fraction of the stable NMD volume displays a sensitivity profile similar to the volatile part<sup>8</sup>. BCBS368 has identified that problem as well but only formulates caps on the maximal fraction of the core within the stable part. All non-core parts shall be treated as O/N deposits.

Nevertheless, a naive replication of the full stable part might lead to a false estimation of the real interest rate risk<sup>9</sup>.

Furthermore, the modelling of NMDs enters the estimation of periodic PnL in NII simulations. Here the behavioural pattern of the run off profile must be adequately taken into account to produce reliable estimates. In practice this means that also a migration of NMDs into other investment types like term deposits should be taken into account.

**Acknowledgment:** The author would like to thank Helge F. Grütjen, Alexander Brunner, Carsta Busch, Carla Mereu and Tilman Wolff-Siemssen for valuable contributions, discussions or corrections, respectively. Any errors left are in the full responsibility of the authors.

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<sup>8</sup>The identification of the investment part belonging to the NMDs compiled by a specific bank becomes a challenge if that bank has a continuously growing business. Therefore, the application of naive statistical analyses onto its customers base might be biased. In that situation one should reconcile the outcome of such an analysis with the outcome of an analysis on the macroeconomical country level, too.

<sup>9</sup>If one uses derivatives to completely hedge any remaining interest rate risk of the combination of banking book positions and modelled NMD positions (without margins and spreads), than the combination of banking book positions and derivatives effectively realise the hypothetical assumed disposition rule of the NMDs. Any inaccuracy in the determination of the run off profile is reflected in a hidden and unhedged interest rate risk.

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